Algorithms and Analysis for Optimizing Robust Objectives in Fair Machine Learning

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What is Welfare-Centric Fair Machine Learning?

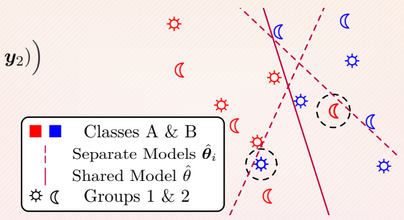
- Fair machine learning considers *multiple groups* $(\boldsymbol{x}_{1:g,1:m}, \boldsymbol{y}_{1:g,1:m})$
- We can handle each group individually
  - *Empirical utility maximization*

$$\hat{\mathrm{U}}(h_\theta; \boldsymbol{x}_i, \boldsymbol{y}_i) \doteq \frac{1}{m}\sum_{j=1}^{m} \mathrm{U}\big(h_\theta(\boldsymbol{x}_{i,j}), \boldsymbol{y}_{i,j}\big); \quad \forall i: \ \hat{\boldsymbol{\theta}}_i \doteq \operatorname*{argmax}_{\theta\in\Theta} \mathrm{U}(h; \boldsymbol{x}_i, \boldsymbol{y}_i)$$

- What is the best classifier *overall?*
  - *Empirical welfare maximization*

$$\hat{\theta} \doteq \operatorname*{argmax}_{\theta\in\Theta} \mathrm{M}\left(\hat{\mathrm{U}}(h_\theta; \boldsymbol{x}_1, \boldsymbol{y}_1), \hat{\mathrm{U}}(h_\theta; \boldsymbol{x}_2, \boldsymbol{y}_2)\right)$$

- Welfare functions encode *social values*
  - Optimize a *given welfare function* $\mathrm{M}(\cdot)$
  - Objectives specify tradeoffs!



Power Means and the Social Planner's Problem

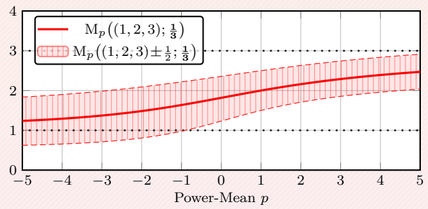
- A *social planner* arranges society to the benefit of all
- How should we aggregate utility or disutility across groups?
- The power-mean for $p \in \mathbb{R}$ summarizes g *values* $\boldsymbol{s}_{1:g}$ with *weights* $\boldsymbol{w}_{1:g}$ as

$$\mathrm{M}_{p\neq 0}(\boldsymbol{s}; \boldsymbol{w}) \doteq \sqrt[p]{\sum_{i=1}^{g} \boldsymbol{w}_i \boldsymbol{s}_i^p}$$

for $p \neq 0$, or

$$\mathrm{M}_0(\boldsymbol{s}; \boldsymbol{w}) \doteq \exp\left(\sum_{i=1}^{g} \boldsymbol{w}_i \ln(\boldsymbol{s}_i)\right)$$

- Fair welfare requires $p \leq 1$, extremes are interesting special cases
  - $p = 1$ is *weighted sum*, a.k.a. utilitarian welfare, over groups (well-studied case)
  - $p = 0$ is the *Nash social welfare* over groups
  - $p = -\infty$ limit is the *minimum* over groups (egalitarian or robust optimization)
- Power-means are:
  1. *Axiomatically Justified*
  2. *Interpretable*
     $\mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w})$ units match $\boldsymbol{s}_{1:g}$
  3. *Stochastically Stable*
     (for $p \in [-\infty, 0) \cup [1, \infty]$)



John Rawls, the Original Position, and the Veil of Ignorance

- Justice, fairness, and societal wellbeing should be *objective concepts*
  - Should not depend on *our own identities*
- The "original position argument"
  - We should assess a situation from behind a "veil of ignorance"
- Rawls argues for *worst-case* robust or pessimistic analysis
  - Egalitarian welfare (malfare) is born!
  - Given utility $\boldsymbol{s} \in \mathbb{R}^g$, assess welfare as $\mathrm{M}_{-\infty}(\boldsymbol{s}) = \min_{i=1}^{g} \boldsymbol{s}_i$
  - Given disutility $\boldsymbol{s} \in \mathbb{R}^g$, assess malfare as $\mathrm{M}_{\infty}(\boldsymbol{s}) = \max_{i=1}^{g} \boldsymbol{s}_i$

This work: We pose the original problem setting as an *adversarial game*

- Robust fair objectives arise as solution concepts against specific adversaries

Egalitarianism and Adversarial Games

The Rawlsian Game

g: *Number of inhabitants* in the game world
Θ: *Parameter space* of worlds Dæmon can create
$\boldsymbol{s}(\theta) : \Theta \to \mathbb{R}^g_{0+}$: (Dis)utility vector of the inhabitants of world θ
$\mathcal{S} \doteq \{\boldsymbol{s}(\theta) \,|\, \theta \in \Theta\} \subseteq \mathbb{R}^g_{0+}$: Set of *feasible (dis)utility vectors* the Dæmon can create
$\mathcal{A}_{\text{Dæ}} \doteq \mathcal{S}$: Dæmon's action space
$\mathcal{A}_{\text{Ang}} \doteq \mathcal{W} \subseteq \triangle_g$: Angel's action space
$\mathrm{P}(\boldsymbol{s}, i) \doteq \pm\langle \boldsymbol{s}_i, -\boldsymbol{s}_i \rangle$: 0-sum payoff function

Strategic gameplay (Dæmon goes first):

$$\arg\min_{\boldsymbol{s}\in\mathcal{A}_{\text{Dæ}}} \max_{i\in\mathcal{A}_{\text{Ang}}} \mathrm{P}_1(\boldsymbol{s}, i) = \arg\min_{\boldsymbol{s}\in\mathcal{S}} \max_{i\in 1,\ldots,g} \boldsymbol{s}_i$$

Weaker Adversaries, Robust Utilitarianism, and the Gini Social Welfare

The Constrained Rawlsian Game

$g, \Theta, \boldsymbol{s}(\theta), \mathcal{S}$: As above
$\mathcal{A}_{\text{Dæ}} \doteq \mathcal{S}$: Dæmon's action space
$\mathcal{A}_{\text{Ang}} \doteq \mathcal{W} \subseteq \triangle_g$: Angel's action space
(*inhabitant* i becomes *distribution* $\boldsymbol{w}$)
New 0-sum payoff function uses *expected (dis)utility:*

$$\mathrm{P}(\boldsymbol{s}, \boldsymbol{w}) \doteq \langle \boldsymbol{w} \cdot \boldsymbol{s}, -\boldsymbol{w} \cdot \boldsymbol{s} \rangle$$

Strategic gameplay (Dæmon goes first):

$$\arg\min_{\boldsymbol{s}\in\mathcal{A}_{\text{Dæ}}} \max_{\boldsymbol{w}\in\mathcal{A}_{\text{Ang}}} \mathrm{P}_1(\boldsymbol{s}, \boldsymbol{w}) = \arg\min_{\boldsymbol{s}\in\mathcal{S}} \max_{\boldsymbol{w}\in\mathcal{W}} \boldsymbol{w} \cdot \boldsymbol{s}$$

A Few Special Cases

- Constant weights $\mathcal{W} = \{\boldsymbol{w}^*\}$: Optimize *utilitarian welfare* $\mathrm{M}_1(\boldsymbol{s}; \boldsymbol{w}^*)$
- Norm ball $\mathcal{W} = \boldsymbol{w}^* + \{\boldsymbol{w} \,|\, \|\boldsymbol{w}\| \leq \gamma\}$: Optimize robust utilitarian welfare
- Weights *bounded from below* $\mathcal{W} \doteq \{\boldsymbol{w} \in \triangle_g \,|\, \boldsymbol{w} \succeq \gamma \boldsymbol{w}^*\}$: Optimize *utilitarian-maximin social welfare function* (UMSWF), i.e.,

$$\min_{\boldsymbol{w}\in\mathcal{W}} \max \mathrm{M}_1(\boldsymbol{s}; \boldsymbol{w}) = \gamma \mathrm{M}_1(\boldsymbol{s}; \boldsymbol{w}^*) + (1-\gamma)\mathrm{M}_{\mp\infty}(\boldsymbol{s})$$

- Given some *sorted weights vector* $\boldsymbol{w}^\downarrow$ or $\boldsymbol{w}^\uparrow$, let $\boldsymbol{s}^\uparrow$ denote *sorted* $\boldsymbol{s}$
  - $\mathrm{M}_{\boldsymbol{w}^\downarrow}(\boldsymbol{s}) \doteq \boldsymbol{w}^\downarrow \cdot \boldsymbol{s}^\uparrow$ is *Gini social welfare*, and $\mathrm{M}_{\boldsymbol{w}^\uparrow}(\boldsymbol{s}) \doteq \boldsymbol{w}^\uparrow \cdot \boldsymbol{s}^\uparrow$ is *Gini social malfare*
  - If Angel has actions $\mathcal{W} \doteq \{\pi(\boldsymbol{w}^\downarrow) | \pi \in \Pi_g\}$, where Π_g is all permutations $\{1, \ldots, g\}$:

$$\max_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_1(\boldsymbol{s}; \boldsymbol{w}) = \mathrm{M}_{\boldsymbol{w}^\downarrow}(\boldsymbol{s}) \quad \text{or} \quad \min_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_1(\boldsymbol{s}; \boldsymbol{w}) = \mathrm{M}_{\boldsymbol{w}^\uparrow}(\boldsymbol{s})$$

A Divine Symmetry: Welfare and Malfare as Game-Theoretic Equilibria

A Game of Dæmonic Justice

- Suppose Dæmon wants to optimize $\begin{matrix}\text{welfare}\\ \text{malfare}\end{matrix}$ fairness concept $\mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w}^*)$
- Angel remains adversarial over some $\mathcal{W}$
- Payoff function

$$\mathrm{P}(\boldsymbol{s}, \boldsymbol{w}) \doteq \big\langle \mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w}), -\mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w}) \big\rangle$$

- Strategic gameplay (Dæmon goes first):

$$\arg\min_{\boldsymbol{s}\in\mathcal{A}_{\text{Dæ}}} \max_{\boldsymbol{w}\in\mathcal{A}_{\text{Ang}}} \mathrm{P}_1(\boldsymbol{s}, \boldsymbol{w}) = \arg\min_{\boldsymbol{s}\in\mathcal{S}} \max_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w})$$

  - Optimizes a *robust power-mean!*

A Game of Angelic Justice

- Suppose Angel wants to optimize $\mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w}^*)$ for some $p > 0$ with action space $\mathcal{W} = \triangle_g$
- We have the payoff function $\mathrm{P}(\boldsymbol{s}, \boldsymbol{w}) \doteq \big\langle \boldsymbol{w} \cdot \boldsymbol{s}, \mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w}^*) \big\rangle$
- Angel strategy: Play $\boldsymbol{w}_i \propto \boldsymbol{w}_i^* \boldsymbol{s}_i^{p-1}$
- Dæmon strategy: Select $\boldsymbol{s}$ to optimize $\boldsymbol{s} \cdot \boldsymbol{w} = \sum_{i=1}^{g} \boldsymbol{w}_i^* \boldsymbol{s}_i^p = \mathrm{M}_p^p(\boldsymbol{s}; \boldsymbol{w})$
- Play at this Nash equilibrium also optimizes a power-mean
- Angel can modify strategy to incorporate *robustness* $\boldsymbol{w}^* \in \mathcal{W}^*$

Fair Learning and Adversarial Optimization

We have reduced our fair and robust fair objectives to the common form

$$\arg\min_{\boldsymbol{s}\in\mathcal{A}_{\text{Dæ}}} \max_{\boldsymbol{w}\in\mathcal{A}_{\text{Ang}}} \mathrm{P}_1(\boldsymbol{s}; \boldsymbol{w}) = \arg\min_{\boldsymbol{s}\in\mathcal{S}} \max_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_p(\boldsymbol{s}; \boldsymbol{w})$$

Lemma 1 (Power-Mean Curvature)
For any $p \geq 1$, if $\boldsymbol{s} : \Theta \to \mathbb{R}^g$ is convex, then $\mathrm{M}_p(\boldsymbol{s}(\theta); \boldsymbol{w}) : (\Theta \times \triangle_g) \to \mathbb{R}_{0+}$ is convex in θ and concave in $\boldsymbol{w}$.
For any $p \leq 1$, if $\boldsymbol{s} : \Theta \to \mathbb{R}^g$ is concave, then $\mathrm{M}_p(\boldsymbol{s}(\theta); \boldsymbol{w}) : (\Theta \times \triangle_g) \to \mathbb{R}_{0+}$ is concave in θ and convex in $\boldsymbol{w}$.

Thus we can efficiently optimize robust fair objectives using *gradient* $\begin{matrix}\text{ascent-descent}\\ \text{descent-ascent}\end{matrix}$ over $\boldsymbol{s} \in \mathcal{S}$

For indirect optimization tasks, we express the task in terms of parameter space Θ as

$$\arg\min_{\boldsymbol{s}\in\mathcal{A}_{\text{Dæ}}} \max_{\boldsymbol{w}\in\mathcal{A}_{\text{Ang}}} \mathrm{P}_1(\boldsymbol{s}; \boldsymbol{w}) = \arg\min_{\theta\in\Theta} \max_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_p(\boldsymbol{s}(\theta); \boldsymbol{w})$$

We can also efficiently optimize robust fair objectives using *gradient* $\begin{matrix}\text{ascent-descent}\\ \text{descent-ascent}\end{matrix}$ over $\theta \in \Theta$

Robust Fair Objectives in Fair Machine Learning

- For any training point x and training label y, the *logistic loss* of model θ is

$$\ell\big(h_\theta(x), y\big) = \ln\left(\frac{1}{1 + \exp(-y \ x \cdot \theta)}\right)$$

- We then compute *disutility* for each group by averaging over training points

$$\boldsymbol{s}_i(\theta) = \frac{1}{\boldsymbol{m}_i}\sum_{j=1}^{\boldsymbol{m}_i} \ell\big(h_\theta(\boldsymbol{x}_{i,j}), \boldsymbol{y}_{i,j}\big)$$

- Observe: $\boldsymbol{s}_i(\theta)$ is *strictly convex* over $\theta \in \Theta$: we can apply maximin methods
- Fair logistic regression objective is then

$$\operatorname*{argmin}_{\theta\in\Theta} \sup_{\boldsymbol{w}\in\mathcal{W}} \mathrm{M}_p\left(i \mapsto \frac{1}{\boldsymbol{m}_i}\sum_{j=1}^{\boldsymbol{m}_i} \ell\big(h_\theta(\boldsymbol{x}_{i,j}), \boldsymbol{y}_{i,j}\big); \boldsymbol{w}\right)$$

Robust Fair Objectives in Fair Allocation

A Most Curious Game

Combining welfare or malfare objectives and robustness yields the *Gini-power-mean family*

$$\mathrm{M}_{\boldsymbol{w}^\downarrow,p}(\boldsymbol{s}) \doteq \mathrm{M}_p(\boldsymbol{s}^\uparrow; \boldsymbol{w}^\downarrow) = \sqrt[p]{\sum_{i=1}^{g} \boldsymbol{w}_i^\downarrow (\boldsymbol{s}_i^\uparrow)^p} \text{ for utility , or}$$

$$\mathrm{M}_{\boldsymbol{w}^\uparrow,p}(\boldsymbol{s}) \doteq \mathrm{M}_p(\boldsymbol{s}^\uparrow; \boldsymbol{w}^\uparrow) = \sqrt[p]{\sum_{i=1}^{g} \boldsymbol{w}_i^\uparrow (\boldsymbol{s}_i^\uparrow)^p} \text{ for disutility}$$

- Generalizes power-mean and Gini families
  - Gini arises for $p = 1$
  - Power-mean (unweighted) for $\boldsymbol{w}^\uparrow = \frac{1}{g}\mathbf{1}$ or $\boldsymbol{w}^\downarrow = \frac{1}{g}\mathbf{1}$
- Arises from power-mean axioms and a robust original position game!